B.sc(H) part 1 paper 1
Topic:The inverse of matrix
Subject mathematics
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The Inverse of Matrix

Defination : If A is a non-singular square matrix, and the matrix B is such that $AB = BA = I$, then B is called the inverse of A and is written as A^{-1}.

It is also called reciprocal matrix.

The analogy in algebra is $a \times \frac{1}{a} = a \times a^{-1} = 1$.

From Art. 4.3, we know that $A \frac{(\text{adj } A)}{|A|} = \frac{(\text{adj } A)}{|A|} A = I$.

Hence we have $B = A^{-1} = \frac{(\text{adj } A)}{|A|}$.

Thus from the preceding theorem $AA^{-1} = I$.

Since A and B are conformal for the product AB and BA and $AB = BA$; it follows that A and B are the square matrices of the same order.

Thus a matrix has an inverse of it only when it is a square matrix.

Theorem

I. Existence of the inverse : Theorem

The necessary and sufficient condition for the existence of the inverse of a square matrix A is that A is non-singular i.e., $|A| \neq 0$.

Proof : *The condition is necessary.*

Let B be the inverse of A.

$\therefore \quad AB = BA = I$

$\therefore \quad |AB| = |I|$

i.e., $\quad |A|\,|B| = 1$

and hence $|A| \neq 0$.

The condition is sufficient.

If $|A| \neq 0$, then $A \cdot \left(\frac{\text{adj } A}{|A|}\right) = I = \left(\frac{\text{adj } A}{|A|}\right) \cdot A$.

so that $B = \frac{\text{adj } A}{|A|}$ is the inverse of A.

Cor. : If A is a non-singular matrix (i.e., $|A| \neq 0$.)

then its inverse $A^{-1} = \frac{\text{adj } A}{|A|}$.

2. The inverse of a matrix is unique.

Proof : Let us suppose that non-singular matrix A has two inverses B and C.

$\therefore \quad AB = BA = I$ and $AC = CA = I$.

$\therefore \quad B = BI = B(AC) = (BA)C = IC = C$

i.e. $B = C$.

Therefore, the inverse of A is unique.

i. Reversal law for the inverse of a product

Theorem : *If A, B be two non-singular matrices of the same order, then* $(AB)^{-1} = B^{-1}A^{-1}$.

We have, $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$ (Associative law)

$= AIA^{-1}; \because \; BB^{-1} = I$

$= AA^{-1}; \because \; AI = A$

$= I.$

Similarly, $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB; \because \; A^{-1}A = I$

$= B^{-1}B; \because \; IB = B$

$= I.$

Thus, $(AB)(B^{-1}A^{-1}) = (B^{-1}A^{-1})AB = I \Rightarrow (AB)^{-1} = B^{-1}A^{-1}$.

order, then $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

In general, if $A, B, C, \ldots K, L$ be non-singular matrices of the same order, then $(ABC \ldots KL)^{-1} = L^{-1}K^{-1}, \ldots C^{-1}B^{-1}A^{-1}$.

Theorem

If A be a non-singular square matrix, then $(A')^{-1} = (A^{-1})'$.

Since $|A'| = |A| \neq 0$, therefore the matrix A' is also non-singular.

We have the identity $AA^{-1} = A^{-1}A = I$.

Now taking the transpose on both sides, we have

$$(AA^{-1})' = (A^{-1}A)' = I'$$

$$\Rightarrow \quad (A^{-1})'A' = A'(A^{-1})' = I;$$

by applying the reversal law for transposes

This shows that $(A^{-1})'$ is the inverse of A'.

Hence $(A')^{-1} = (A^{-1})'$.

i.e., the inverse of the transpose of a matrix is the transpose of the inverse.

Cor. : If the non-singular matrix A is symmetric, then A^{-1} is also symmetric.

Since A is symmetric, therefore $A' = A$.

Now, we have $(A^{-1})' = (A')^{-1}$

$$= A^{-1}, \text{ since } A' = A.$$

This shows that A^{-1} is symmetric.